Dynamic State Networks

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Cellular Automata (CA)

- Why are cellular automata interesting?
- What do we want from the study of CA?
- Why should an alternative be introduced?

Using Graphs

-Unifying complex networks and complex dynamics

-Well-developed mathematical framework

-More general modeling of complex systems

A dynamic state network (DSN) is defined as a pair (G,R). Let the graph G = (V,E) comprise set of vertices V and edges E. Every v_i in V contains a state, t. Let a rule R be defined as a mapping from neigborhoods of radius r, N_r, to a states t_n.

Implementation issues

-Nonuniform neighborhoods

-Redefining rules

-Qualifying and quantifying complexity

Redefining rules

-Uniform neighborhoods are not guaranteed
-To work around this, let us define the relative totalistic rule. Instead of mapping a total to a state, we can map the ratio between the total and $N_r \cdot t_{max}$ to a state.

Qualifying and quantifying complexity

-Two types of complexity: structural and behavioral
-Structural complexity deals with the topology of the DSN
-Behavioral complexity concerns the temporal evolution of the DSN

Entropy as a measure of complexity

$-\Sigma$ p log p taken over an arbitrary probability distribution produced by counting frequencies of elements

-In a cellular automata, frequencies are taken over neighborhoods

-Generalizing entropy from cellular automata to DSNs is not straightforward

Isomorphism entropy

-Direct generalization of CA entropy

-Counting neighborhoods requires equivalence classes for graph neighborhoods

-Use isomorphism classes, where two graphs, G and H, are isomorphic if there exists a bijection f: $\mathrm{V}(G) \rightarrow \mathrm{V}(H)$, such that (u,v) is in $\mathrm{E}(G)$ iff $(f(u),f(v))$ is in $\mathrm{E}(H)$ and $G_s(u) = H_s(f(u))$

Isomorphism Entropy (cont'd)

-For any non vertex-transitive graph, the minimum isomorphism entropy is not necessarily zero
-For some graphs, the maximum isomorphism entropy is equal to the minimum entropy
-For graphs with a small diameter, neighborhood sizes grow very quickly with radius
-Isomorphism is not known to be in P.

Arc entropy

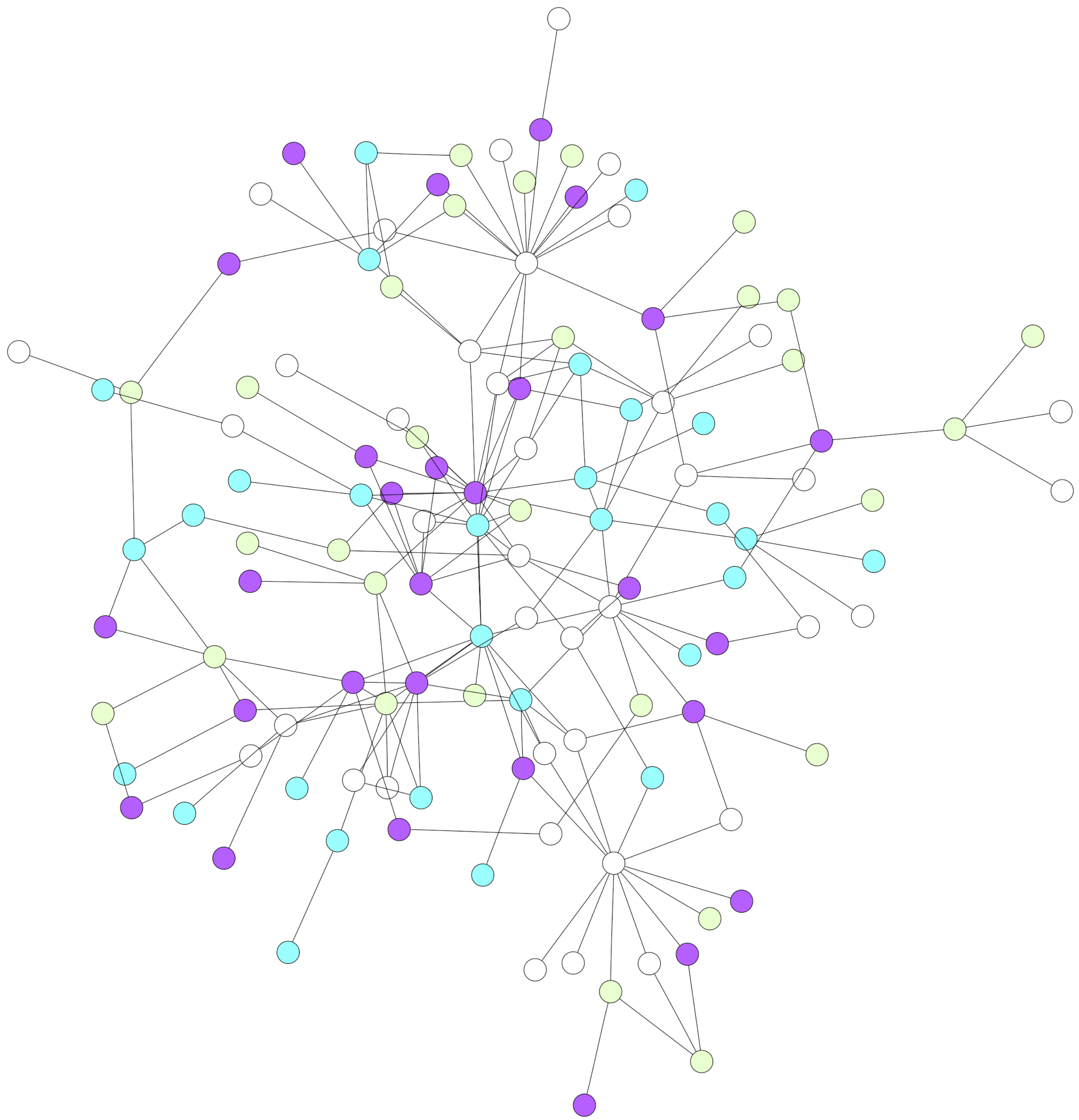
-Entropy is taken over all arcs of length q

-An arc is an acyclic, nonempty subgraph of G whose vertices $v_1..v_n$ are connected by edges $\{(v_i,v_{i+1})\}$

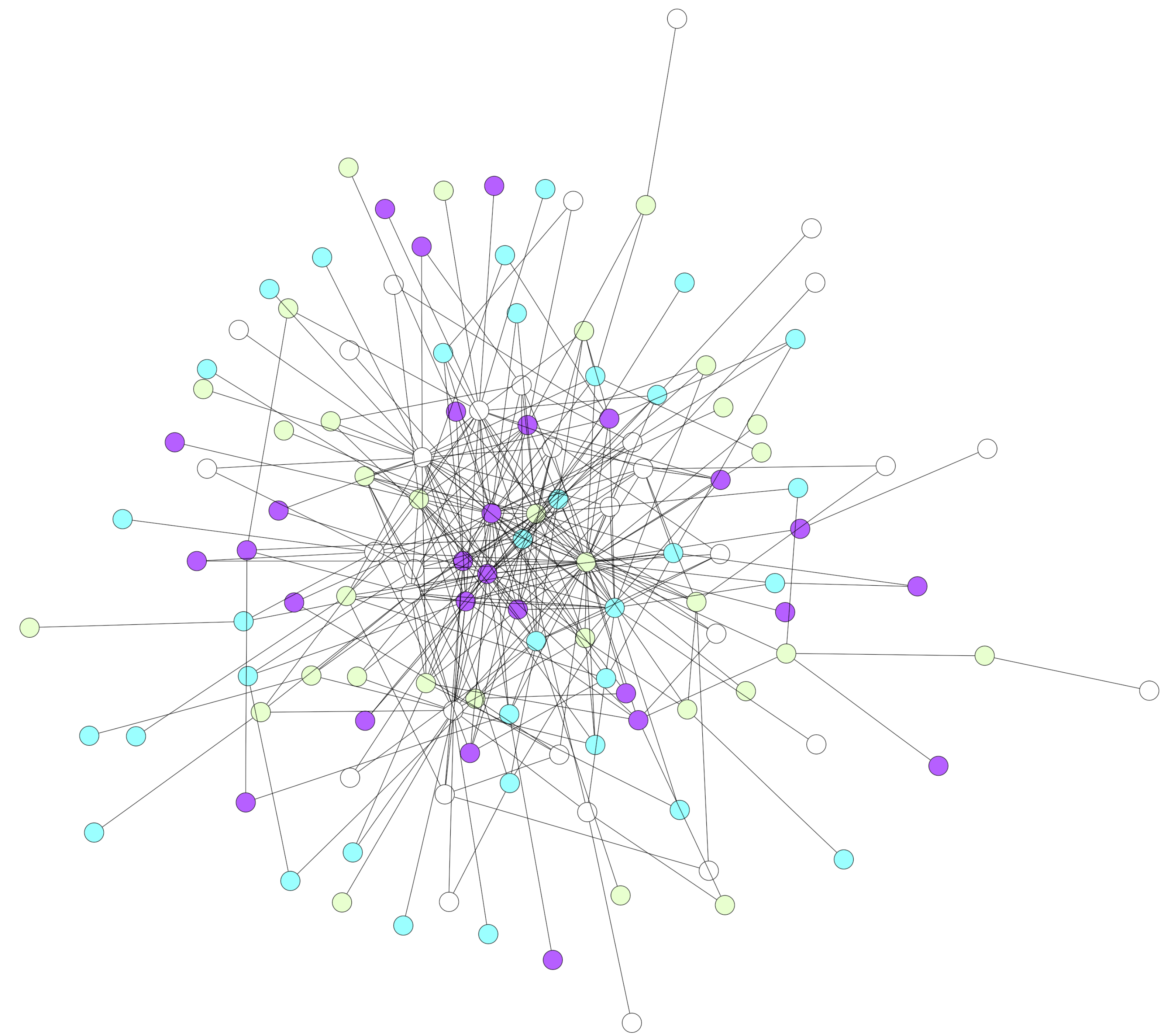
-Two arcs are equal if $G_s(v_i) = G_s(u_i)$ for all i

Arc entropy (cont'd)

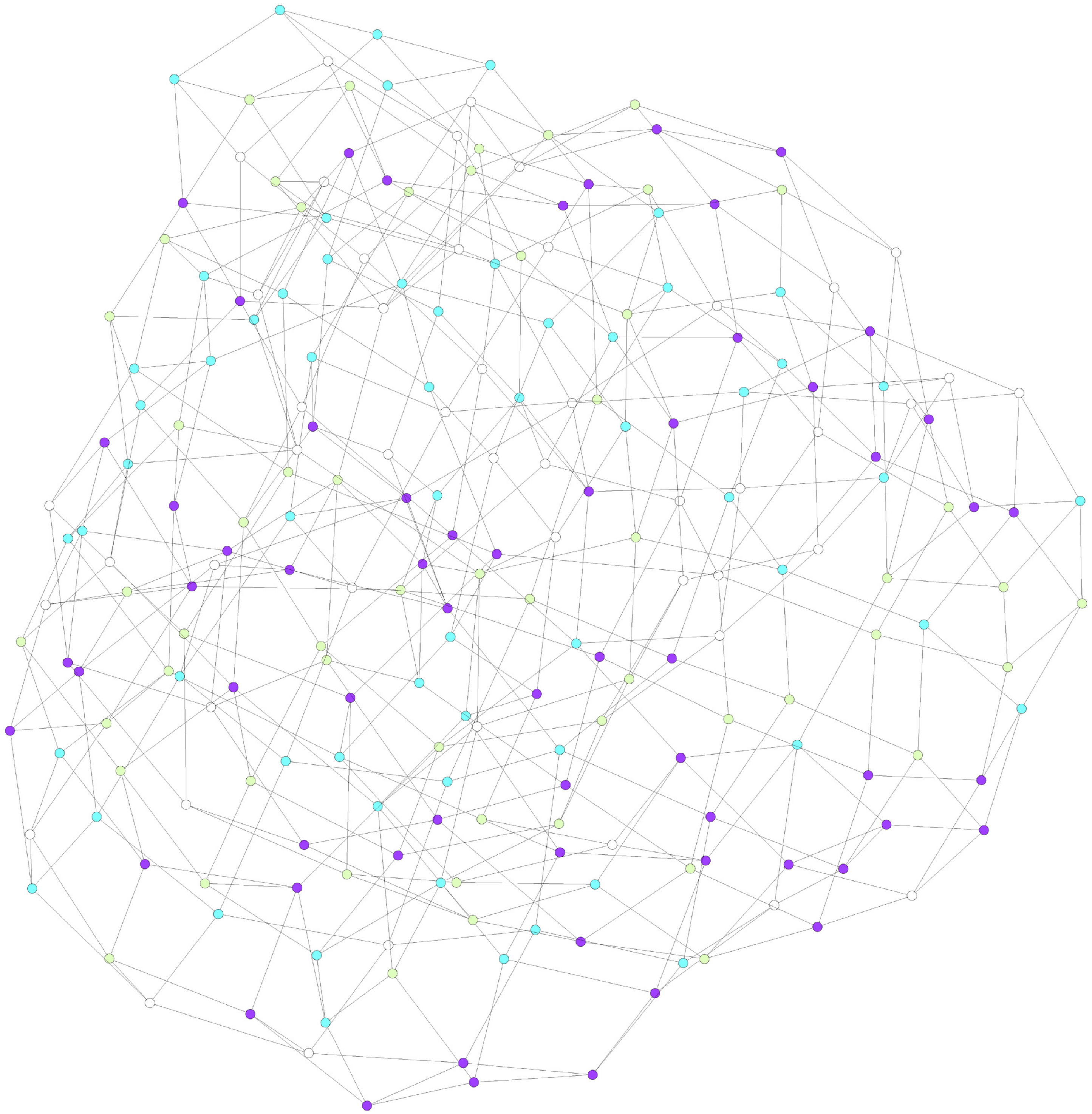
-Solves some of the problems with isomorphism entropy
-Loses some of the structural information about a graph in reducing a neighborhood to one-dimensional arcs
-Runs in $O(n!/(n-q+1)!)$ in the worst case, a complete graph



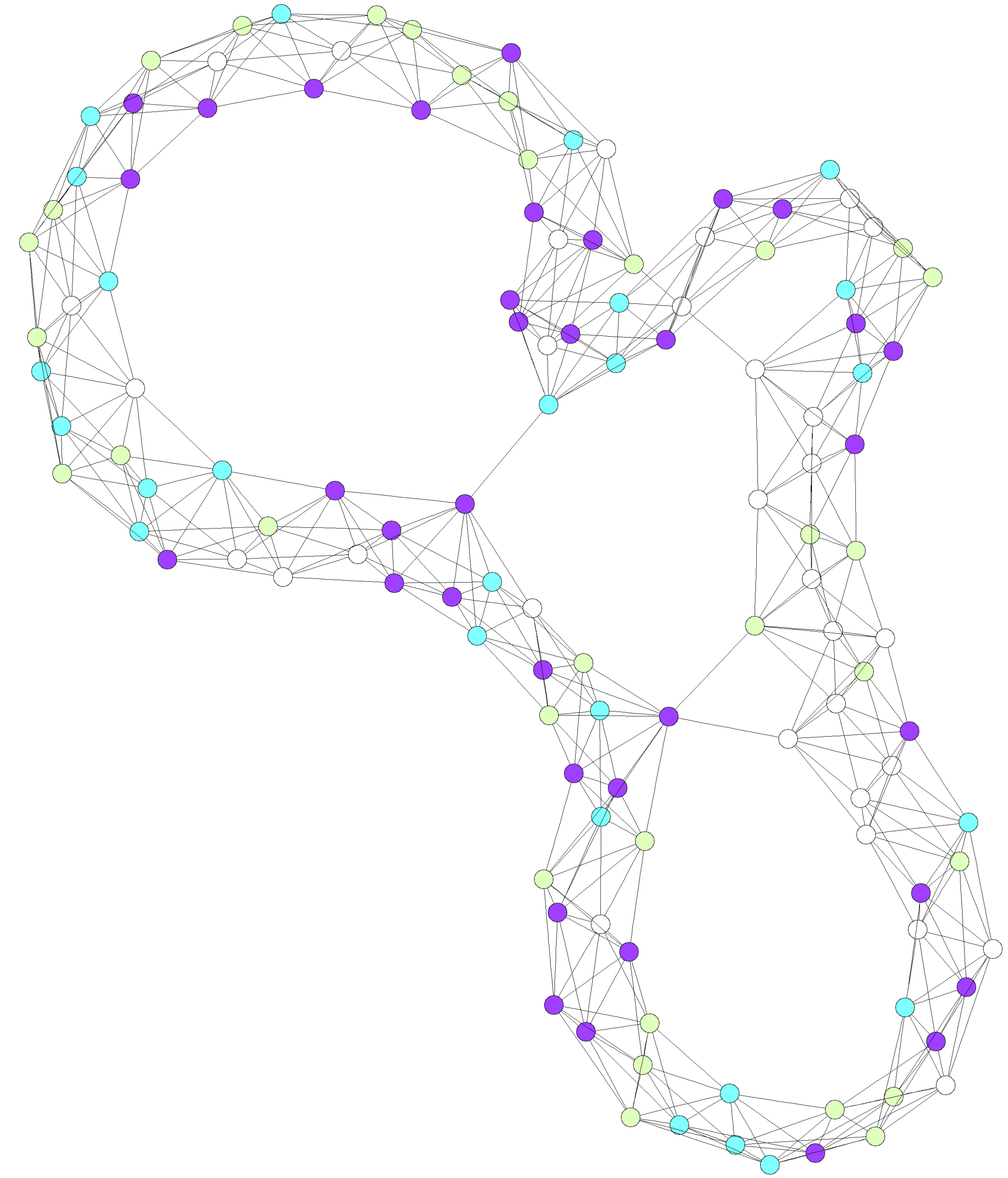
Vertex duplication



Preferential attachment



Grid exponential

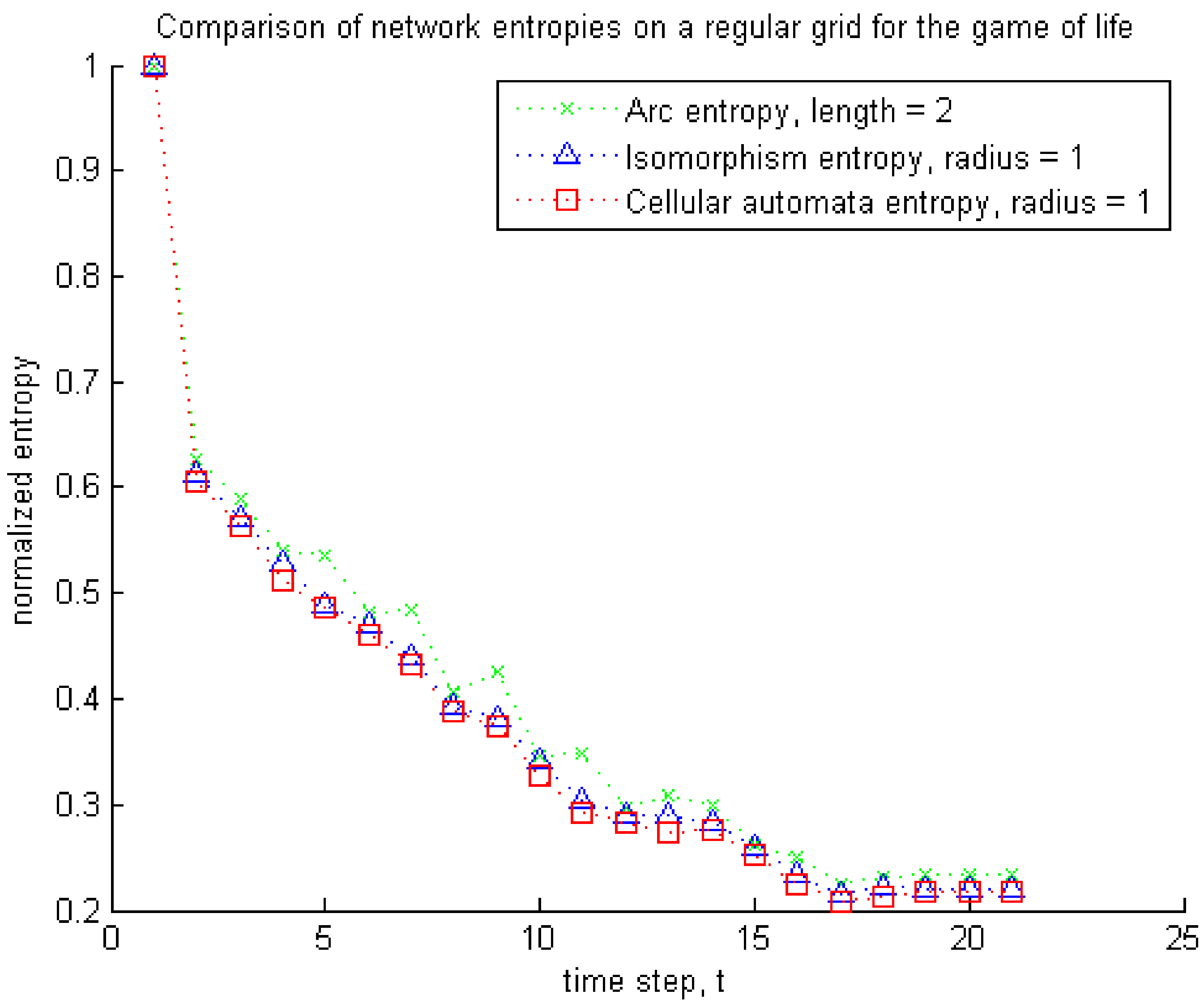


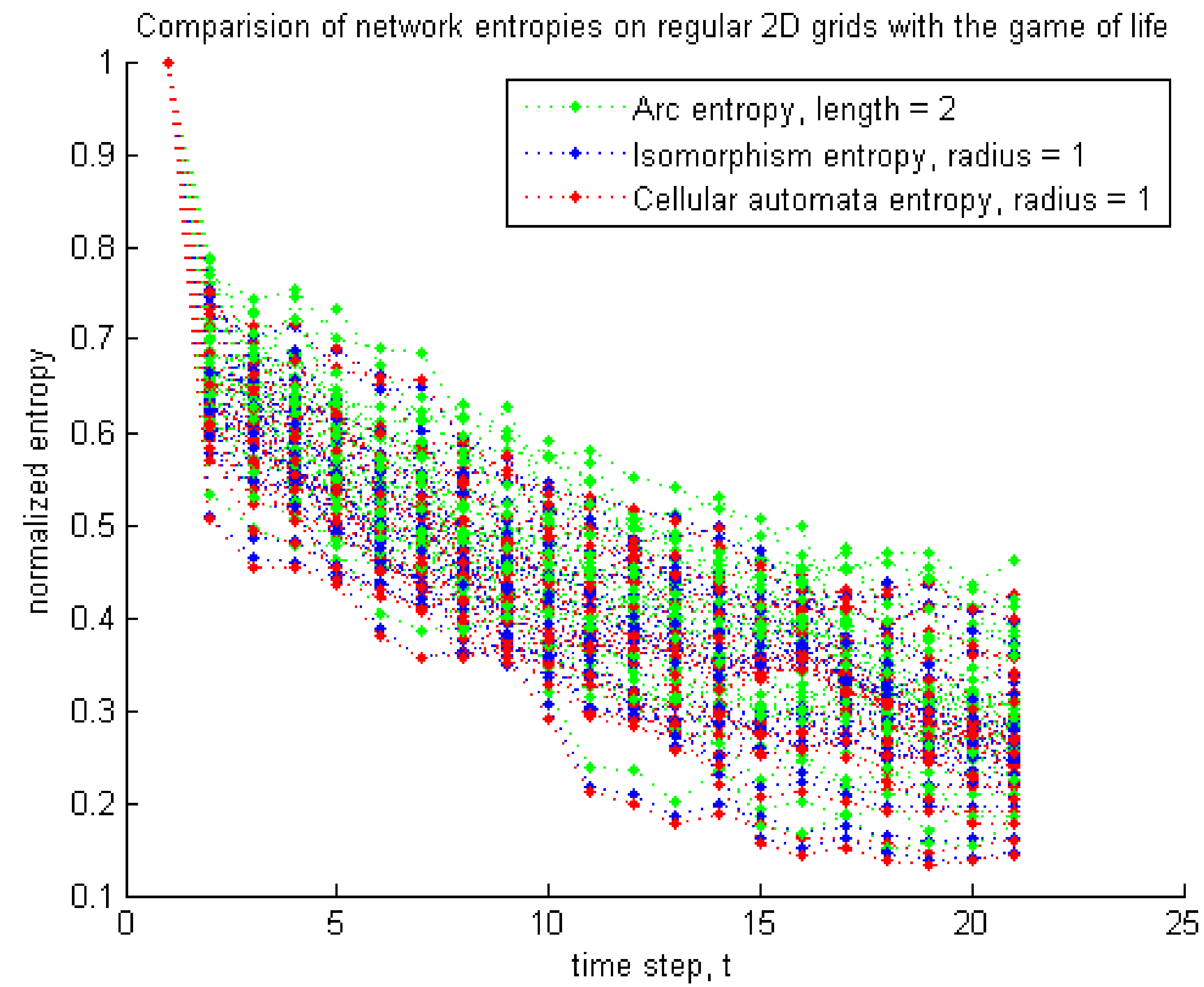
Ring exponential

Relative “Game of Life” rule

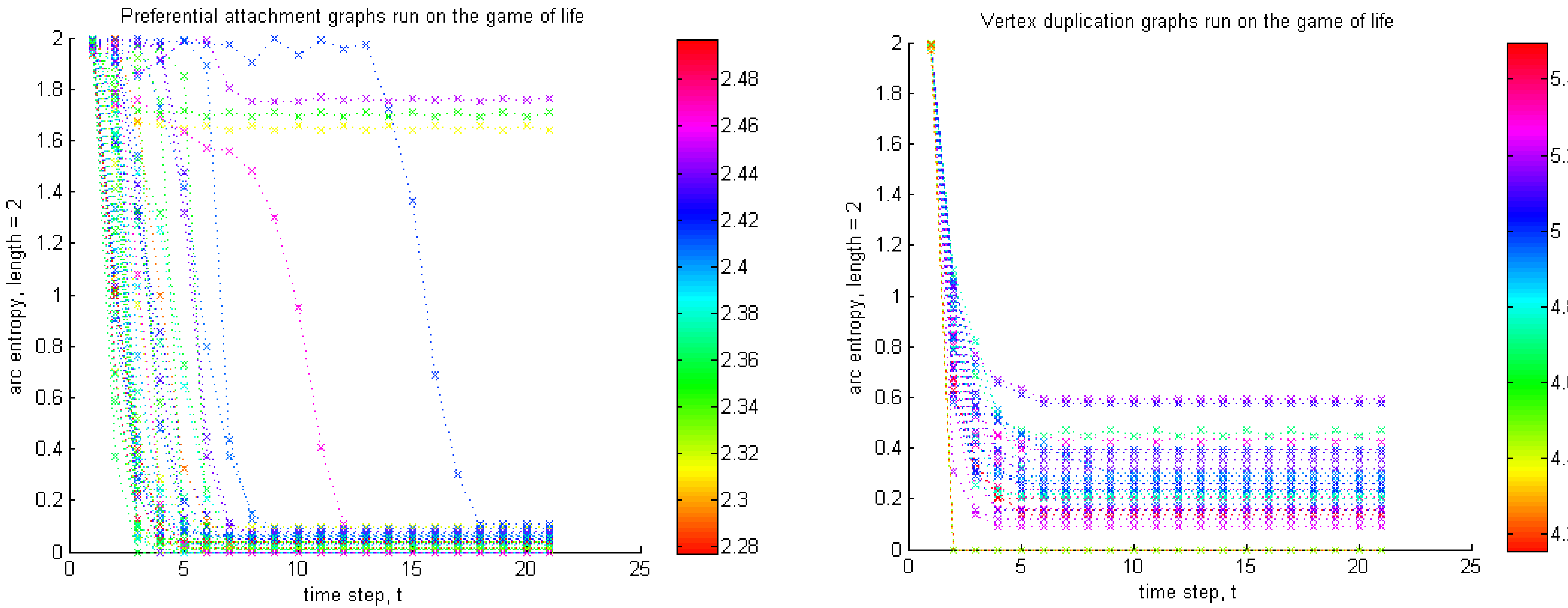
Current state	Ratio,p, of alive neighbors	Next state
alive	0 <= p < 0.25	dead
alive	0.25 <= p < .5	alive
alive	0.5 <= p <= 1.0	dead
dead	0 <= p < 0.375	dead
dead	0.375 <= p < 0.5	alive
dead	0.5 <= p < 1.0	dead

Comparing the new entropies to the old

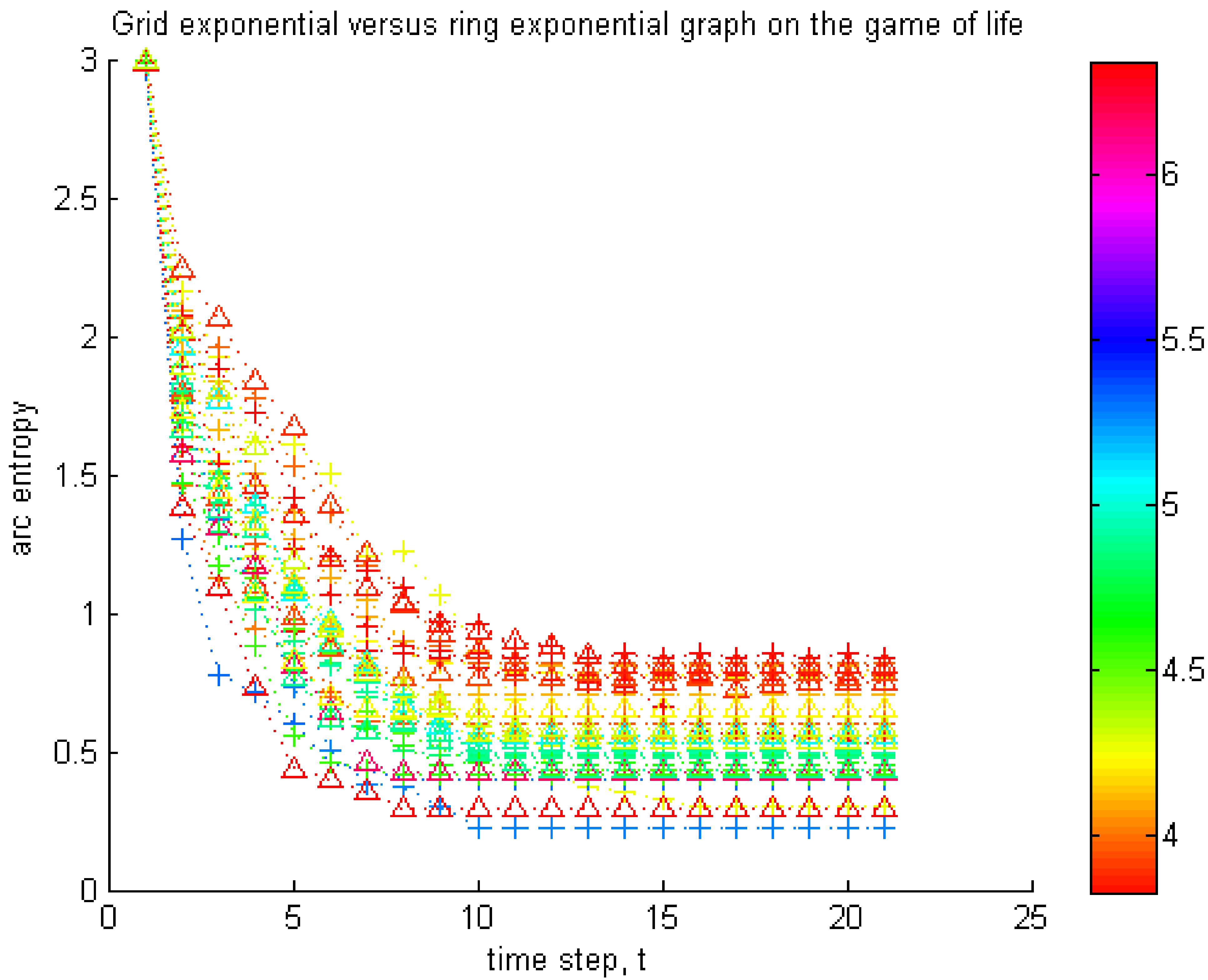




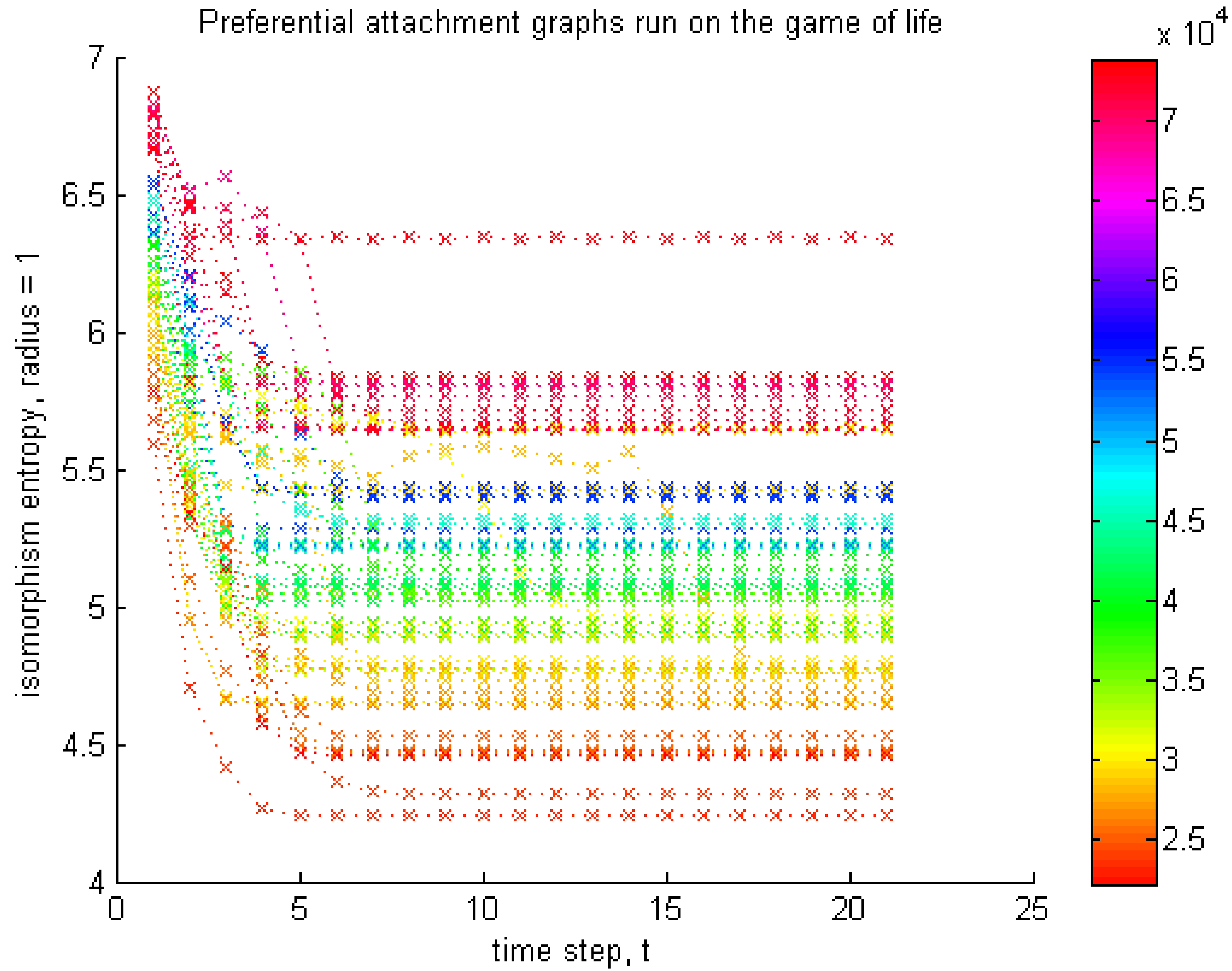
Comparing power law graph: arc entropy



Comparing exponential graphs: arc entropy



Isomorphism entropy



Conclusions

-The DSN provides a novel, powerful framework for both the empirical and theoretical investigation of complex systems

-We have introduced entropy measures on the DSN analagous to those on their CA counterparts

-Preliminary results indicate that the topology of the network on which a rule is run materially affects the behavior of the DSN.

-Potential correspondence between particular graph properties and DSN behavior.

Future Work

-Expanding framework to include directed, propertied edges
-Extensive, if not comprehensive, exploration of the DSN rule space
-Concretize the relationship between the behavior of the DSN, its topological properties, and the rules placed upon it.
-Develop new methods of analysis that take advantage of the explicit structure of conditionality in the DSN